Spectral-Energy Efficiency Tradeoff in Full-Duplex Two-Way Relay Networks

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Abstract—Owing to its high spectral efficiency, two-way relaying has aroused tremendous research interests. Recently, substantial progress in self-interference cancellation makes full-duplex two-way relaying practical. For this new paradigm, analyzing spectral-energy efficiency tradeoff is crucial, which has not been addressed in the existing works in the literature. In this paper, the spectral-energy efficiency tradeoff in a full-duplex two-way relay network with amplify-and-forward relaying is studied by considering the residual self-interference at the relay. An optimization problem is formulated to maximize the energy efficiency under the spectral efficiency requirement and the maximum transmission power constraints by adjusting the transmission powers of the terminals and the amplification gain of the relay. A lower-complexity iterative optimization algorithm is developed to solve the optimization problem. Simulation results show that: 1) the proposed algorithm can achieve optimal energy efficiency which is consistent to the one obtained by the exclusive searching method; 2) the full-duplex two-way relay network can achieve higher spectral efficiency but lower optimal energy efficiency compared to the half-duplex one; 3) the optimal energy efficiency is insensitive to the residual power of self-interference, when the relay is located near either terminal.

Index Terms—Full-duplex, two-way relaying, self-interference, spectral efficiency, energy efficiency.

I. INTRODUCTION

With the promotion of green ICT, energy-efficient wireless communications become more and more attractive. Meanwhile, spectral efficiency (SE) is another metric in wireless communications which has been widely investigated in the past. Unfortunately, SE and energy efficiency (EE) are not always consistent and sometimes even have a mutual contradiction [1], [2]. Therefore, balancing the tradeoff between SE and EE will have a vital significance for future wireless communications.

Cooperative relaying has been envisaged as a spectral- and energy-efficient technology for cellular networks [3] and future 5G mobile communication systems [4]. It can enhance received signal-to-noise ratio (SNR) and bit error rate (BER), extend communication ranges, and reduce transmission power consumption. Thus, the exploitation of this technique is expected to dramatically improve the capacity, reliability, coverage, and EE of wireless networks.

Based on the operation mode of relays, relaying systems can be categorized into two types: half-duplex (HD) [5] and full-duplex (FD) [6], [7]. In literature, a large number of works have been done on HD relaying with focus on SE [8], channel capacity [9], outage probability [10], etc. However, in HD relaying, the source-to-relay link and the relay-to-destination link are implemented in different time slots or different frequency bands, which causes degradation on SE. On the contrary, FD relaying allows the relay nodes to receive and transmit data at the same time in the same frequency band. Obviously, FD relaying can achieve up to twice of the capacity of HD relaying. Although FD relaying has great potential over HD relaying in terms of SE, it is confronted with self-interference (SI) due to simultaneous transmission and reception at the relay node. Recently, some practical SI cancellation schemes have been developed for FD transmissions [11]–[14], which prompt the research work in this area. For example, low-complexity precoders for maximizing both SE and EE in an FD multiuser multiple-input multiple-output system was proposed in [15]. The EE aspect of resource allocation in an OFDMA cellular network was considered in [16], where a shared FD relay was deployed at the intersection of three sectors of a cell. The problem of joint bandwidth sharing and power allocation was formulated as a three-stage Stackelberg game. However, in all these works, discussions on SE-EE tradeoff in FD relay networks were missed.

With respect to the directions of signal transmission, relaying systems can be classified into three types called one-way, two-way, and multi-way. Among them, one-way relaying has been extensively studied for both HD and FD modes [15]–[18]. Compared with one-way relaying, two-way relaying (TWR) can achieve higher SE, which allows a relay to simultaneously communicate with two end nodes. However, existing work on TWR mainly focused on the HD mode, while little work has been done on the FD mode, let alone study the SE-EE tradeoff in TWR network with FD mode.

It is well known that full-duplex two-way relaying can significantly enhance spectral efficiency, but its energy efficiency has not been demonstrated. In this paper, we consider a full-duplex two-way (FDTW) relay network in which two terminals communicate with each other with the assistance of an FD relay. The tradeoff between SE and EE in the
FDTW relay network with amplify-and-forward (AF) relaying is studied. We take into account SI at the relay and maximize the EE under the constraints on SE and the transmission power budget. After formulating an optimization problem, we propose an iterative optimization algorithm to find the solution with guaranteed convergence. The contributions of this paper are summarized as follows:

1) From the perspective of practical implementation, SI cannot be fully canceled due to channel estimation error. Differing from existing works on FDTW relay networks, we consider remaining SI after cancellation at the relay in a more reasonable and relatively tractable way to study the tradeoff between SE and EE in the FDTW relay network with AF relaying;

2) The formulated optimization problem is non-convex. To overcome this issue, the primal optimization problem is first decomposed into two sub-problems, and then we solve the two sub-problems alternatively. For the first sub-problem, we show that there exists an equivalent non-fractional form, which can be effectively solved by the Dinkelbach method [19]. For the second sub-problem, we first prove it is convex and then solve it by the bisection method;

3) Through computer simulations, the SE and the optimal EE obtained by the proposed iterative optimization algorithm are evaluated in the FDTW relay network. Besides, the comparison between the FDTW relay network and the half-duplex two-way (HDTW) relay network in terms of achievable SE and EE is made. Finally, we demonstrate that the proposed algorithm can converge very quickly.

The remainder of this paper is organized as follows. In Section II, the related work is summarized. In Section III, the FDTW relay network model is described and the tradeoff between SE and EE is formulated as an optimization problem. The solving process of the optimization problem is elaborated and an iterative optimization algorithm is presented in Section IV. Simulation results are illustrated in Section V and finally concluding remarks are drawn in Section VI.

II. RELATED WORK

In the literature, some works have been done on EE of TWR systems. For example, a three-node TWR system with digital network coding was considered in [20]. The aim was to minimize the total energy consumption while ensuring queue stability at all nodes for given random packet arrival rates. To offer high EE, a hybrid one-way and two-way relay strategy was proposed in [21], where the number of bits and transmission time allocated to the one-way and two-way relaying stages were jointly optimized to minimize the overall energy consumption. The EE of a two-way decode-and-forward relay system with three nodes was studied in [22]. The maximization of EE was achieved by jointly optimizing the transmission time and power of each node, with an assumption of non-ideal power amplifiers. Energy-efficient beamforming was studied for a multiple-input multiple-output two-way non-regenerative relaying system with multi-antenna users and a multi-user relay [23]. In [24], the power allocation problem was solved in the context of maximizing EE of OFDM signaling over a two-way AF relay network. There are a few works on the tradeoff between SE and EE in HDTW relaying systems. For instance, the relationship between SE and EE in a two-way multiple relay system with an analog network coding was investigated in [25]. The SE-EE tradeoff in a two-way relaying system with amplify-and-forward strategy was investigated in [26], where statistical channel state information (CSI) was required and the asymmetric traffic requirements were considered. However, to the best of our knowledge, there is no work addressing SE–EE tradeoff in FDTW relay networks, which is the emphasis of this paper.

III. SYSTEM MODEL

Consider an FDTW relay network, where a source node (SN) and a destination node (DN) communicate with each other under the assistance of a relay node (RN), as shown in Fig. 1. SN and DN both transmit their signals to RN, and at the same time, RN broadcast the previous received signals to SN and DN. Each node operates in the FD mode and equips a single transmitting antenna and a single receiving antenna. There is no direct link between SN and DN (e.g., due to shadowing).

Assume that channels between SN and RN as well as between RN and DN are both reciprocal. The complex channel coefficient between SN and RN, and between RN and DN, denoted by $h_{sr}$, $h_{rd}$, are zero-mean, circularly symmetric complex Gaussian (CSCG) random variables with variances $\sigma_{sr}^2 = d_{sr}^{-\nu}$, $\sigma_{rd}^2 = d_{rd}^{-\nu}$, respectively. Herein the subscripts $s, r, d$ represent SN, RN, DN, respectively, $d_{ij}$ is the normalized distance between nodes $i$ and $j$, and $\nu$ is the path-loss exponent. The effective channel gains are defined as $G_{ij} = |h_{ij}|^2$. We consider quasi-static fading in which the channel coefficients are constant within one defined frame, but may change independently from one frame to another. Therefore, at any time in a frame $t$, the received signal at RN can be expressed as [23, 24]

$$y_{r}(t) = \sqrt{P_s}h_{sr}x_{s}(t) + \sqrt{P_d}h_{rd}x_{d}(t) + \sqrt{P_r}h_{rr}r(t) + N_{r}(t),$$

where $x_{s}(t)$ and $x_{d}(t)$ are the transmitted signals of SN and DN, respectively, with $\mathbb{E}\{|x_{s}(t)|^2\} = \mathbb{E}\{|x_{d}(t)|^2\} = 1$. Here $\mathbb{E}\{x\}$ defines the expectation of the random variable $x$, $P_s$ and $P_d$ are the transmission powers of SN and DN, respectively. Unlike some previous works, we consider SI in the FDTW relay network which can be reduced but cannot be completely eliminated by an interference cancellation technique. The existence of SI will make the optimization problem formulation complicated as shown later. $\rho_s$ denotes the average residual power of SI after cancellation at RN and has been modeled as an invariable value. There are two main reasons for us to
employ this model. One is that if \( \rho_s \) is regarded as a function of the transmission power of RN, both the primal problem (P) and the transformed sub-problem (P1) (defined in Section III and Section IV-A, respectively) may become non-convex. The second reason is that the power of SI can be controlled in a range after cancellation [7]. Thus, it is reasonable to regard \( \rho_s \) to be fixed\(^1\). The issue that \( \rho_s \) may be associated with the transmission power of RN will be considered in our future works. The SI channel coefficient at RN is denoted by \( h_{rr} \), which is a zero-mean, CSCG random variable with variance \( \sigma_{rr}^2 \) (Experimental results in [27] showed that after applying a sufficiently large analog domain cancellation, the strong line-of-sight component is attenuated so that the residual SI follows a Ricean distribution with a small \( K \)-factor or a Rayleigh distribution). \( N_r(t) \) is an additive white Gaussian noise (AWGN) with zero mean and variance \( \sigma_r^2 \).

RN adopts an AF protocol to broadcast the received signal from both SN and DN at the time instant \( t - 1 \), i.e., \( r(t) = y_r(t-1) \), where \( r(t) \) is the broadcasted signal of RN at the time instant \( t \). To meet the transmission power constraint of RN, the average power of \( r(t) \) is normalized to 1. We assume that SN and DN own full CSI while RN only owns channel gain information. It is because the full CSI is required by SN and DN to implement perfect interference cancellation, and the channel gain information is sufficient for RN to implement the SE–EE tradeoff. Hence, the received signals after subtracting interference at SN and DN are respectively given by

\[
\begin{align*}
y_s(t) &= \sqrt{\beta} h_{sr} r(t) + N_s(t) = \sqrt{\beta P_{rd}} h_{sr} h_{rd} x_r(t-1) \\
&+ \sqrt{\beta} h_{hr} h_{rr} r(t-1) + \sqrt{\rho_s} N_r(t-1) + N_s(t), \quad (2)
y_d(t) &= \sqrt{\beta} h_{rd} r(t) + N_d(t) = \sqrt{\beta P_{sr}} h_{sd} h_{rd} x_r(t-1) \\
&+ \sqrt{\rho_r} h_{rd} h_{rr} r(t-1) + \sqrt{\rho_d} N_r(t-1) + N_d(t), \quad (3)
\end{align*}
\]

where \( \beta \) is the amplification gain of RN which should satisfy the constraint \( 0 \leq \beta \leq \beta^* \) with \( \beta^* = \frac{\eta_{SE \max} + P_{max} \eta_{EE \max} + \sigma_r^2}{\sigma_d^2 \beta^*} \). \( P_{max} \) is the maximum allowed transmission power of RN, and \( N_s \) and \( N_d \) are AWGNs with zero mean and variance \( \sigma^2 \).

According to (2) and (3), the output signal-to-interference-plus-noise ratios (SINRs) at SN and DN can be respectively calculated as

\[
\begin{align*}
\gamma_S &= \frac{\beta P_{rd} G_{sr} G_{rd}}{\beta \rho_s G_{sr} G_{rr} + \beta G_{sd} \sigma_s^2 + \sigma^2}, \quad (4) \\
\gamma_D &= \frac{\beta P_{sr} G_{rd} G_{rd}}{\beta \rho_r G_{rd} G_{rr} + \beta G_{rd} \sigma_d^2 + \sigma^2}. \quad (5)
\end{align*}
\]

Hence, the overall SE is

\[
\eta_{SE}(P_s, P_d, \beta) = R_s + R_d, \quad (6)
\]

where \( R_s = \log_2(1 + \gamma_s) \) and \( R_d = \log_2(1 + \gamma_d) \). Here the channel bandwidth has been normalized to 1 Hz. The system EE can then be defined as

\[
\eta_{EE}(P_s, P_d, \beta) = \frac{\eta_{SE}(P_s, P_d, \beta)}{P_T(P_s, P_d, \beta)} \text{ [bits/Joule]}, \quad (7)
\]

\(^1\)Although the instantaneous power of SI \( \rho_s(t) \) after cancellation may vary over time, in this paper, we use \( \rho_s \), as aforementioned, to denote the average power of SI.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>COMMONLY USED ABBREVIATIONS AND NOTATIONS</th>
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<tbody>
<tr>
<td>Abbreviation/Notation</td>
<td>Interpretation</td>
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<tr>
<td>SE</td>
<td>Spectral efficiency</td>
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<tr>
<td>EE</td>
<td>Energy efficiency</td>
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<tr>
<td>HD</td>
<td>Half-duplex</td>
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<tr>
<td>FD</td>
<td>Full-duplex</td>
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<td>SI</td>
<td>Self-interference</td>
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<td>FDTW</td>
<td>Full-duplex two-way</td>
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<td>AF</td>
<td>Amplify-and-forward</td>
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<tr>
<td>HDTW</td>
<td>Half-duplex two-way</td>
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<tr>
<td>CS</td>
<td>Channel state information</td>
</tr>
<tr>
<td>SN</td>
<td>Source node</td>
</tr>
<tr>
<td>DN</td>
<td>Destination node</td>
</tr>
<tr>
<td>RN</td>
<td>Relay node</td>
</tr>
<tr>
<td>G_{i,j}</td>
<td>Channel gain between node ( i ) and ( j )</td>
</tr>
<tr>
<td>P_s</td>
<td>Transmission power of SN</td>
</tr>
<tr>
<td>P_d</td>
<td>Transmission power of DN</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>Average residual power of self-interference</td>
</tr>
<tr>
<td>( \sigma_s )</td>
<td>Noise variance</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Amplification gain of RN</td>
</tr>
<tr>
<td>( \beta^* )</td>
<td>Maximum allowed amplification gain of RN</td>
</tr>
<tr>
<td>( \eta_{SE} )</td>
<td>Overall spectral efficiency</td>
</tr>
<tr>
<td>( \eta_{EE} )</td>
<td>System energy efficiency</td>
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<tr>
<td>( \eta_{SE \min} )</td>
<td>Minimum required spectral efficiency</td>
</tr>
</tbody>
</table>

\( P_s \) and \( P_d \) are the maximum allowed transmission powers of SN and DN, respectively.\(^2\)

\[^2\]where \( P_T(P_s, P_d, \beta) \) is the total power consumption which is calculated by \( P_T(P_s, P_d, \beta) = P_s + P_d + \beta(P_s G_{sr} + P_d G_{rd} + \rho_s G_{rr} + \sigma_s^2) + P_c \), and \( P_c \) denotes the power consumption in the circuitry and is regarded as a constant. For notation simplicity, in the following, SE and EE may be represented as \( \eta_{SE} \) and \( \eta_{EE} \), respectively, without confusion.

Our objective is to seek the optimal \( P_s, P_d, \beta \) to maximize \( \eta_{EE} \) under the constraints on \( \eta_{SE} \) and the maximum transmission power. The optimization problem can be formulated as

\[
\begin{align*}
\max_{P_s, P_d, \beta} & \quad \eta_{EE} \\
\text{s.t.} & \quad \eta_{SE} \geq \eta_{SE \min} \geq 0, \\
& \quad 0 \leq P_s \leq \overline{P}_s, \\
& \quad 0 \leq P_d \leq \overline{P}_d, \\
& \quad 0 \leq \beta \leq \beta^*,
\end{align*}
\]

where \( \eta_{SE} \) is the minimum required SE, and \( \overline{P}_s \) and \( \overline{P}_d \) are the maximum allowed transmission powers of SN and DN, respectively. For notational clarity, the commonly used abbreviations and notations are summarized in Table I.

IV. SE–EE Tradeoff

In the optimization problem (P), both \( \eta_{EE} \) and \( \eta_{SE} \) are non-convex with respect to \( P_s, P_d, \) and \( \beta \), so that directly solving problem (P) introduces enormous computational complexity. To overcome this issue, a new iterative optimization algorithm is proposed in this section. The basic idea is to first optimize the objective function over a portion of variables when the others are fixed, and then searches all the potential results to produce the optimal solution. Specifically, we first optimize the problem (P) over \( P_s \) and \( P_d \) with a fixed \( \beta \), and then optimize the problem (P) over \( \beta \) with fixed \( P_s \) and \( P_d \). This process will be repeated until convergence.
A. Optimization over \( P_s \) and \( P_d \) with a Fixed \( \beta \)

Given a fixed \( \beta \), our objective is to seek the optimal transmission powers \( P_s \) and \( P_d \) to maximize \( \eta_{EE} \) while satisfying the required constraints. Thus, given \( \beta \), the problem (P) can be rewritten as

\[
\max_{P_s, P_d} \eta_{SE} \frac{P_s}{P_d}
\]

s.t. \( \eta_{SE} \geq \eta_{SE}^{\star} \geq 0, \quad 0 \leq P_s \leq \mathcal{T}_s, \quad 0 \leq P_d \leq \mathcal{T}_d. \) \hspace{1cm} (P1)

**Proposition 1:** For any given \( \beta \), \( \eta_{SE} \) is a concave and strictly increasing function of \( P_s \) and \( P_d \) for \( P_s, P_d \in [0, +\infty) \).

**Proof:** Refer to Appendix A.

Although the constraints in the sub-problem (P1) are convex according to **Proposition 1**, the objective function is still non-convex [28]. To address this issue, we apply the fractional programming technique [19] to transform the objective function to a concave one.

**Proposition 2:** The sub-problem (P1) achieves the maximum value \( q^\star = \frac{\eta_{SE}^{\star}}{P^\star_{T}} \) as long as \( \max_{P_s, P_d} \{ \eta_{SE}(P_s, P_d) - qP_T(P_s, P_d) \} = \eta_{SE}(P^\star_{s}, P^\star_{d}) - qP_T(P^\star_{s}, P^\star_{d}) = 0 \) and for a given \( q \), the transformed function in subtractive form is a concave function.

**Proof:** Refer to Appendix B.

According to **Proposition 2**, we can conclude that the solution of sub-problem (P1) is equivalent to \( \max_{P_s, P_d} F(q^\star) = 0 \), where \( F(q^\star) = \eta_{SE}(P^\star_{s}, P^\star_{d}) - qP_T(P^\star_{s}, P^\star_{d}) \). Thus, there exists a non-fractional expression which is equal to a fractional function. Searching \( q^\star \) can be done by the Dinkelbach method which can converge to the optimal value at a superlinear rate [19]. Therefore, for any feasible \( q \), the sub-problem (P1) can be equivalently transformed into a sub-problem (P2), as follows:

\[
\max_{P_s, P_d} \eta_{SE}(P_s, P_d) - qP_T(P_s, P_d)
\]

s.t. \( \eta_{SE} \geq \eta_{SE}^{\star} \geq 0, \quad 0 \leq P_s \leq \mathcal{T}_s, \quad 0 \leq P_d \leq \mathcal{T}_d. \) \hspace{1cm} (P2)

Since the sub-problem (P2) is a convex optimization problem for a given \( q \), we can derive the optimal solution in an effective way.

The partial Lagrangian function of the sub-problem (P2) can be expressed as

\[
L_{EE}(P_s, P_d, \lambda) = \eta_{SE}(P_s, P_d) - qP_T(P_s, P_d) + \lambda(\eta_{SE} - \eta_{SE}^{\star}),
\]

where \( \lambda \) is the Lagrangian multiplier associated with the inequality constraints and is required to be non-negative. By using the Karush-Kuhn-Tucker (KKT) conditions, it is required that the first-order derivative of \( L_{EE} \) with respect to \( P_s \), \( P_d \), and \( \lambda \) be zero, i.e.,

\[
(1 + \lambda) \frac{\eta}{\eta P_s + \psi} = q \ln(1 + \beta G_{sr}), \quad (9)
\]

\[
(1 + \lambda) \frac{\eta}{\eta P_d + \theta} = q \ln(1 + \beta G_{rd}), \quad (10)
\]

\[
\lambda \bigg( \frac{q P_d + \eta}{P_d} + \frac{\eta^2}{\psi \theta} P_s \bigg) - \varpi = 0, \quad (11)
\]

where \( \eta = \beta G_{sr} G_{rd}, \quad \psi = \beta \rho_s G_{rd} G_{rr} + \beta G_{rd} \sigma^2 + \sigma^2, \quad \theta = \beta \rho_s G_{sr} G_{rr} + \beta G_{sr} \sigma^2 + \sigma^2, \quad \varpi = 2 \eta_{SE} - 1. \)

From (9) and (10), we can obtain

\[
P^\star_s = \left[ \frac{1 + \lambda}{q \ln(1 + \beta G_{sr})} - \frac{\psi}{\eta} \right]^+, \quad P^\star_d = \left[ \frac{1 + \lambda}{q \ln(1 + \beta G_{rd})} - \frac{\theta}{\eta} \right]^+, \quad (12)
\]

where \([x]^+\) denotes \( \max(0, x) \) which guarantees \( P_s \geq 0 \) and \( P_d \geq 0 \).

Notice that the optimal transmission powers of SN and DN given by (12) are similar to customized water-filling solutions, where the heights of the pool are defined as \( \frac{\psi}{\eta} \) and \( \frac{\theta}{\eta} \), and the water level is partially determined by \( \lambda \) and \( q \). Because both \( P_s \) and \( P_d \) involve the Lagrangian multiplier \( \lambda \), and the KKT conditions of (8) require \( \lambda \) to be non-negative, we derive \( \lambda \) in the following two cases.

- **Case 1:** \( \lambda > 0 \)

In this case, from (11), we have \( \frac{\eta}{\psi} P_d + \frac{\eta^2}{\psi \theta} P_s P_d - \varpi = 0 \). After substituting (12), we have \( (\lambda + 1)^2 S_1 - S_2 = 0 \), where \( S_1 = \frac{\eta^2}{\eta \psi \theta (1 + \beta G_{sr}) (1 + \beta G_{rd})} \) and \( S_2 = \varpi + 1. \)

Since \( \lambda > 0 \), only the positive root is retained, i.e., \( \lambda^* = \sqrt{S_2} \) \( \frac{1}{S_1} - 1 \). Substituting this positive root back to (12) yields the optimal solution of the sub-problem (P2) as

\[
P^\star_s = \left[ \frac{\sqrt{S_2}}{q \ln(1 + \beta G_{sr})} - \frac{\psi}{\eta} \right]^+, \quad P^\star_d = \left[ \frac{\sqrt{S_2}}{q \ln(1 + \beta G_{rd})} - \frac{\theta}{\eta} \right]^+. \quad (13)
\]

- **Case 2:** \( \lambda = 0 \)

In this case, (11) always holds true. Thus, we have

\[
P^\star_s = \left[ \frac{1}{q \ln(1 + \beta G_{sr})} - \frac{\psi}{\eta} \right]^+, \quad P^\star_d = \left[ \frac{1}{q \ln(1 + \beta G_{rd})} - \frac{\theta}{\eta} \right]^+. \quad (14)
\]

Note that, \( P^\star_s \) and \( P^\star_d \) in (14) may break the SE constraint. If this happens, the solution (13) should be adopted. Otherwise, both solutions (13) and (14) are feasible and the one leading to greater EE is the final solution. Considering the transmission power constraints, the final solution can be expressed as \( P_s = \min (\mathcal{T}_s, P^\star_s), P_d = \min (\mathcal{T}_d, P^\star_d) \). Since the transformed problem (P2) is convex with a given \( q \), the optimal EE can be obtained with a given \( \beta \) by the Dinkelbach method. According to the above analysis, the detailed procedure of the Dinkelbach method is listed in Table II.

**Remark 1:** Since iterations are needed in the process of solving the sub-problem (P1) by applying the Dinkelbach
method, the initial value of $q$ should be properly set. In order to let the initial value $q^{(0)}$ be in the feasible region of Case 1, $q^{(0)}$ needs to satisfy $q^{(0)} > \frac{\ln 2}{\sqrt{8 \epsilon (\sum_{i=1}^{s} + 1) (1 + \beta G_{sr}) (1 + \beta G_{rd})}}$. However, $q^{(0)}$ cannot be too large, which may make $\eta_{SE}(P_s, P_d) - q^{(0)} P_T(P_s, P_d)$ be less than zero [19].

### B. Optimization over $\beta$ with Fixed $P_s$ and $P_d$

In this sub-problem, our objective is to maximize $\eta_{EE}$ while satisfying the minimum required $\eta_{SE}$ and the constraint of $\beta$, which can be formulated as

$$\max_{P_s, P_d} \eta_{EE} \quad \text{s.t.} \quad \eta_{SE} \geq \eta_{SE}^0, \quad 0 \leq \beta \leq \beta^*.$$  \hspace{1cm} (P3)

It is easy to prove that $\eta_{SE}$ is an increasing function of $\beta \in [0, +\infty)$ and the domain of the function $\eta_{SE}$ (i.e. $\eta_{SE} \geq 0$) and the constraint $\eta_{SE} \geq \eta_{SE}^0$, we have $\beta \in \mathbb{R}$, where $\sum_i = \{\beta/\beta \in [-\infty, \beta^*] \cup [\beta^*, +\infty]\}$, $\beta^* = \frac{b-b_0}{2a}$, and $a = (P_s G_{sr} G_{rd} G_{rr} + G_{sr} G_{rd} G_{rr}^2)(P_d G_{rd} + P_s G_{sr}) + P_s P_d G_{sr}^2 G_{rd}^2 - c$, $b = G_{sr} G_{rd} (P_s + P_d) \sigma^2 - c$, $d_1 = (\rho P_s G_{sr} G_{rd} + G_{sr} \sigma^2) (\rho P_d G_{rd} + G_{rd} \sigma^2)$, $d_2 = (\rho P_s G_{sr} G_{rd} + G_{sr} \sigma^2) (\rho P_d G_{rd} + G_{rd} \sigma^2)$, and $c = -\sigma^4$.

Since $\beta \in [0, \beta^*]$ and $\beta \in \mathbb{R}$, the feasible region of the sub-problem (P3) becomes $\sum_i = [\beta, \beta^*]$.

**Proposition 3:** Given $P_s, P_d \in [0, +\infty)$, $\eta_{EE}$ is a strictly quasi-concave function of $\beta$ for $\beta \in [0, +\infty]$.

**Proof:** Refer to Appendix C.

From **Proposition 3**, we have

- **Case 1**
  - If $\eta_{EE}$ is strictly increasing with $\beta \in \sum_2$ (i.e., $\frac{d\eta_{EE}}{d\beta} > 0$), the optimal solution to the sub-problem (P3) is achieved at $\beta = \beta^*$.

- **Case 2**
  - If $\eta_{EE}$ is strictly decreasing with $\beta \in \sum_2$ (i.e., $\frac{d\eta_{EE}}{d\beta} < 0$), the optimal solution to the sub-problem (P3) is achieved at $\beta = \beta^*$.

### V. Simulation Results

The EE-SE tradeoff in the aforementioned FDTW relay network is simulated by using the Matlab. The maximum transmission powers of SN and DN are set to $P_s = 4$ dBW.
and $P_d = 4$ dBW, respectively, and the maximum transmission power of RN is set to $P_{r_{max}} = 5$ dBW. The circuitry power consumption is set to $P_c = 2$ dBW. The required minimum SE is set to $\eta_{SE} = 2$ bps/Hz. All the variances are normalized to one, i.e., $\sigma^2_{rr} = \sigma^2 = 1$ W. We set $v = 4$, $d_{sd} = 1$ m, and $d_{rd} = d_{sd} - d_{sr}$. In order to compare the attainable SE or EE in HDTW and FDTW relay networks, and investigate the impact of residual power of SI on SE and EE in the FDTW relay network, both SE and EE are evaluated in the following. The results are obtained by averaging over 10 000 different channel realizations.

A. Effectiveness of the Proposed Algorithm

For evaluating the proposed method, we first compare the proposed algorithm with the exclusive searching method when $\rho_r = 3$ dBW in Figs. 2 and 3. From these figures, we can observe that the results obtained by the iterative optimization algorithm are very close to those obtained by exclusive searching, which indicates that our proposed algorithm can reach a sub-optimal solution. In addition, as shown in Figs. 2 and 3, we can observe that the optimal EE will first increases and then decreases after reaching the peak value at $d_{sr} = 0.5$ m while SE satisfies the constraint which also first increases and then decreases. The main reason is that as shown in Fig. 4, the increase of SE contributes to the increase of EE when the SE is less than 3.5 bps/Hz. From Fig. 3, we can see that the maximum SE in our simulation is about 3 bps/Hz when the relay node locates at the middle of SN and DN. Thus, by combing these two observations, we can conclude that the optimal EE reach the peak value at the middle of SN and DN. In addition, from Fig. 5, we can observe that the amplification gain $\beta$ reaches maximum when RN is located at the middle of SN and DN as well. Therefore, we can claim that the largest transmission power is used under this situation.

The exclusive searching needs to evaluate all the possible values of $P_s, P_d, \beta$, so that it needs $O(\frac{P_s P_d}{\rho_r} \times \frac{\beta - \beta}{\epsilon})$ steps to get the optimal value, where $\epsilon$ is the searching accuracy. In the proposed algorithm, both the bisection method and the Dinkelbach method are applied alternatively, thus the total steps is $O(M(N + \log_2(\frac{\beta - \beta}{\epsilon})))$, where $M$ and $N$ are the average iterative steps of the outer loop and Dinkelbach method respectively, and are both set to 10 in our simulation. Thus the proposed method has significantly reduced computational complexity compared with the exclusive searching method.

B. Relationship Between EE and SE

In this subsection, we investigate the relationship between the optimal EE and the required SE (i.e. $\eta_{SE}$) as well as the obtained SE (i.e. $\eta_{SE}$) which calculated by the proposed
Fig. 6. Average EE versus $\eta_{SE}$.

algorithm when RN is located in $d_{sr}$=0.2, 0.5 and 0.8 m separately, and $\rho_r = 2$ dBW. As shown in Fig. 4, when $\eta_{SE} \leq 3.5$ bps/Hz, the optimal EE will increase with $\eta_{SE}$. However, when $\eta_{SE} > 3.5$ bps/Hz, the optimal EE will decrease with increasing $\eta_{SE}$. Meanwhile, from Fig. 6, even though it describes the optimal EE versus the obtained SE, we can still get the same conclusions as from Fig. 4. This phenomena can be simply explained as follows. When the required SE is below a certain threshold, the optimal EE will mainly be affected by the SE. When the required SE is above this threshold, the optimal EE will be dominated by the total transmission power, which leads to decreased EE.

C. Comparison Between FDTW and HDTW Relay Network on EE and SE

Figs. 7 and 8 show the comparison between the FDTW relay network and the HDTW relay network on optimal EE and SE, respectively. From Fig. 7, it can be seen that the HDTW relay network achieves higher optimal EE than the FDTW relay network, while the SE of the FDTW relay network is much higher than that of the HDTW relay network as shown in Fig. 8. These phenomena can be intuitively explained as follows. Although the FDTW relay network can achieve higher SE compared to the HDTW relay network, the FD transmission mode works in two time slots simultaneously compared to HD [17] and needs to overcome SI which leads to more power consumption than the HD transmission mode. Therefore, EE of the FDTW relay network decreases.

D. Effect of SI on EE and SE

Figs. 9 and 10 illustrate the effect of residual power of SI on EE and SE in the FDTW relay network. As depicted in Figs. 9 and 10, the achieved optimal EE and SE values will decrease with the increase of residual power of SI. Meanwhile, when the distance between RN and SN or that between RN and DN is very short, such as $d_{sr} \leq 0.1$ m or $d_{sr} \geq 0.9$ m, the residual power of SI has little effect on both EE and SE. This is because when RN is close to SN, the transmission power of DN, $P_{d}$, dominates $\eta_{SE}$ so that the effect of the residual SI power at the RN becomes marginal.

E. Convergence of the Proposed Algorithm

Fig. 11 illustrates the convergence of the proposed optimization algorithm in terms of average optimal EE under random channel realizations. The RN is located in the middle of the
line between the SN and DN (i.e., $d_{sr} = 0.5$ m). It can be seen from Fig. 11 that the proposed algorithm can converge only after 3 iterations.

**VI. CONCLUSIONS**

In this paper, the SE and EE tradeoff in an FDTW relay network with AF relaying has been investigated. By taking SI at RN into account, we formulated an optimization problem to maximize the EE under both the SE constraint and the transmission power constraints. Since the primal optimization problem (P) is non-concave with respect to the variables $P_s$, $P_d$, and $\beta$, we decomposed it into two equivalent sub-problems and devised an iterative optimization algorithm to get the solution. For the sub-problem (P1) that is optimized over $P_s$ and $P_d$ for a fixed $\beta$, a concave-convex fractional programming technique and the Lagrange multiplier method were successively applied for solution. For the sub-problem (P3), which optimizes over $\beta$ for fixed $P_s$ and $P_d$, we first proved that $\eta_{SE}$ is a quasi-concave function of $\beta$ and then seek the solution by using the bisection method. Simulation results indicate that the proposed algorithm can achieve nearly the same EE as the exclusive searching method. The FDTW relay network can achieve higher SE than the HDTV relay network, at the cost of lower optimal EE. Furthermore, the optimal EE is insensitive to the residual power of SI, when RN is located near SN or DN. These results are helpful for the design of spectral- and energy-efficient FDTW relay networks.

Future work can be done according to the following two aspects: 1) In this paper, $p_r$ is regarded to be fixed. It is meaningful to build a more practical model for $p_r$ and investigate the SE-EE tradeoff accordingly; 2) The theoretical results are derived under the assumption of perfect CSI. However, CSI cannot be perfectly estimated in practice. It is imperative to study the SE-EE tradeoff under imperfect CSI.

**APPENDIX A**

**PROOF OF PROPOSITION 1**

We take the first-order partial derivative of $\gamma_s$ and $\gamma_d$ with respect to $P_s$ and $P_d$ respectively as

$$\frac{\partial \gamma_s}{\partial P_s} = \frac{\beta G_{sr} G_{rd}}{\ln 2 (\beta G_{sr} G_{rd} + \beta G_{sr} \sigma^2 + \sigma^2)^2} > 0, \quad \frac{\partial \gamma_d}{\partial P_d} = \frac{\beta G_{sr} G_{rd}}{\ln 2 (\beta G_{sr} G_{rd} + \beta G_{sr} \sigma^2 + \sigma^2)^2} > 0,$$

Then from (6), the first and second partial derivative of $\eta_{SE}$ with respect to $P_s$ and $P_d$ can be calculated as

$$\frac{\partial \eta_{SE}}{\partial P_s} = \frac{1}{\ln 2} \frac{\beta G_{sr} G_{rd}}{(\beta G_{sr} G_{rd} + \beta G_{sr} \sigma^2 + \sigma^2)^2} > 0,$$

$$\frac{\partial \eta_{SE}}{\partial P_d} = \frac{1}{\ln 2} \frac{\beta G_{sr} G_{rd}}{(\beta G_{sr} G_{rd} + \beta G_{sr} \sigma^2 + \sigma^2)^2} > 0,$$

$$\frac{\partial^2 \eta_{SE}}{\partial P_s^2} = -\frac{1}{\ln 2} \frac{\beta G_{sr} G_{rd}}{(\beta G_{sr} G_{rd} + \beta G_{sr} \sigma^2 + \sigma^2)^2} < 0,$$

$$\frac{\partial^2 \eta_{SE}}{\partial P_d^2} = -\frac{1}{\ln 2} \frac{\beta G_{sr} G_{rd}}{(\beta G_{sr} G_{rd} + \beta G_{sr} \sigma^2 + \sigma^2)^2} < 0,$$

$$\frac{\partial^2 \eta_{SE}}{\partial P_s \partial P_d} = -\frac{\partial^2 \eta_{SE}}{\partial P_s \partial P_d} = 0,$$

$$\frac{\partial^2 \eta_{SE}}{\partial P_s^2} \frac{\partial^2 \eta_{SE}}{\partial P_d^2} - \frac{\partial^2 \eta_{SE}}{\partial P_s \partial P_d} \frac{\partial^2 \eta_{SE}}{\partial P_s \partial P_d} > 0.$$

From (17)–(20), we can conclude that the Hessian matrix of $\eta_{SE}$ is negatively definite. Thus, $\eta_{SE}$ is a concave function for $P_s, P_d \in [0, +\infty)$. In addition, from (15) and (16), $\eta_{SE}$ is strictly increasing with $P_s$ or $P_d$. The proof of Proposition 1 is complete.

**APPENDIX B**

**PROOF OF PROPOSITION 2**

Define $F(q) = \eta_{SE}(P_s, P_d) - q P_r T(P_s, P_d)$. Proposition 2 means i) solution to the sub-problem (P1) is equivalent to $\max_{P_s, P_d} F(q^*) = 0$, and ii) $F(q)$ is convex for any given $q$. We prove the equivalence and convexity separately.

The proof of equivalence is divided into two parts: necessity and sufficiency. We first prove the necessity. Suppose that the maximum EE of the sub-problem (P1) is $q^* = \frac{\eta_{SE}(P_s, P_d)}{P_r T(P_s, P_d)}$. 

![Fig. 10. Comparison of average SE for different residual power of SI.](image)

![Fig. 11. Convergence of the proposed algorithm in terms of average optimal EE ($p_r = 2$ dBW).](image)
Then we have the following inequality for any $P_s$ and $P_d$ in the feasible region:

$$q^* = \frac{\eta_{SE}(P_s^*, P_d^*)}{P_T(P_s^*, P_d^*)} \geq \frac{\eta_{SE}(P_s, P_d)}{P_T(P_s, P_d)} \implies \eta_{SE}(P_s, P_d) - q^* P_T(P_s, P_d) \leq \eta_{SE}(P_s^*, P_d^*) - q^* P_T(P_s^*, P_d^*) = 0. \quad (21)$$

From (21), the maximum value of EE can be achieved when $\eta_{SE}(P_s, P_d) - q^* P_T(P_s, P_d)$ is equal to zero, which can be achieved at $P_s = P_s^*$ and $P_d = P_d^*$. This completes the proof of necessity.

Next, we prove the sufficiency. Suppose that $\max_{P_s, P_d} \eta_{SE}(P_s, P_d) - q^* P_T(P_s, P_d) = \eta_{SE}(P_s^*, P_d^*) - q^* P_T(P_s^*, P_d^*) = 0$. Then, for any given $P_s$ and $P_d$ in the feasible region, we have

$$\eta_{SE}(P_s, P_d) - q^* P_T(P_s, P_d) \leq \eta_{SE}(P_s^*, P_d^*) - q^* P_T(P_s^*, P_d^*) = 0. \quad (22)$$

From (22), we have

$$\frac{\eta_{SE}(P_s, P_d)}{P_T(P_s, P_d)} \leq \frac{\eta_{SE}(P_s^*, P_d^*)}{P_T(P_s^*, P_d^*)} = q^*. \quad (23)$$

Thus, the maximum value of EE can be achieved at $P_s = P_s^*$, $P_d = P_d^*$ and we have proved the sufficiency. Thus, optimal solution to the sub-problem (P1) and the problem $\max_{P_s, P_d} F(q^*) = 0$ are equivalent.

Now we prove the convexity. For a given $q$, the transformed subtractive form can be written as $\eta_{SE}(P_s, P_d) - q P_T(P_s, P_d)$. The first part of the subtractive form is $\eta_{SE}(P_s, P_d)$ which is a concave function in $P_s, P_d \in [0, +\infty)$ and the second part is an affine function. Since the sum of a concave function and an affine function is also a concave function, the proof of convexity is complete.

**APPENDIX C**

**PROOF OF PROPOSITION 3**

We first introduce the following equations and inequality:

$$a_1 = P_s G_{ss} + P_d G_{sd} + P_r G_{sr} + \sigma^2, \quad a_2 = P_s + P_d + P_r,$$

$$c_1 = P_d G_{ss} + P_d G_{sd} + P_r G_{sr} + G_{sr} \sigma^2,$$

$$c_2 = P_s G_{sr} + G_{sr} \sigma^2,$$

$$c_3 = P_s G_{sr} + G_{sr} \sigma^2,$$

$$k_1 = c_1 c_4 + c_1 c_3 + c_2 c_4 + c_2 c_3, k_2 = (c_1 + c_2 + c_3 + c_4) \sigma^2,$$

$$k_3 = c_2 c_4, k_4 = k_2 + k_3, \quad k_2^2 - k_2 - k_3 < 0, k_3 - k_1 < 2, 2 \sigma^2 (k_1 - k_3) + k_2^2 - k_2 < 0.$$

From (7), the first-order derivative of $\eta_{EE}$ with respect to $\beta$ is

$$\frac{d\eta_{EE}}{d\beta} = \frac{(a_1 + a_2)(2k_1 \beta + k_2 - k_3 \beta + k_4 - 2k_3 \beta + k_4)}{\ln 2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2)} - \frac{a_1 \log_2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2)}{\ln 2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2)}$$

$$- (a_1 + a_2)^2 (2k_1 \beta + k_2 + k_3 \beta + \sigma^2)$$

$$\implies f(\beta) = \frac{(a_1 + a_2)(2k_1 \beta + k_2 + k_3 \beta + \sigma^2) - (2k_1 + k_2) + 2k_3 \beta + k_4)}{(k_1 \beta + k_2 + k_3 \beta + \sigma^2)^2}$$

$$- \frac{a_1 \log_2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2)}{\ln 2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2)} \leq 0$$

$$< 0, \quad \beta \in [0, +\infty). \quad (25)$$

Then the first-order derivative of $f(\beta)$ with respect to $\beta$ can be calculated as

$$\frac{df(\beta)}{d\beta} = \frac{(a_1 + a_2)(2k_1 \beta + k_2 + k_3 \beta + \sigma^2) - (2k_1 + k_2) + 2k_3 \beta + k_4}{(k_1 \beta + k_2 + k_3 \beta + \sigma^2)^2}$$

$$- \frac{a_1 \log_2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2)}{\ln 2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2)} \leq 0$$

$$< 0, \quad \beta \in [0, +\infty). \quad (26)$$

Hence, we have $f(\infty) < f(0) < f(0), \forall \beta \in [0, +\infty)$. Since $\lim_{\beta \to +\infty} f(\beta) = -a_1 \log_2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2) > 0$ and $\lim_{\beta \to +\infty} f(\beta) = -a_1 \log_2 (k_1 \beta + k_2 + k_3 \beta + \sigma^2) > 0$, there exists a single value of $\beta$ denoted as $\beta^*$, so that $f(\beta^*) = 0$. Obviously, since the denominator of (25) is positive, we have $\frac{d\eta_{EE}}{d\beta} > 0$ when $\beta > \beta^*$ and $\frac{d\eta_{EE}}{d\beta} < 0$ when $\beta < \beta^*$. It means that $\eta_{EE}$ first increases and then decreases when $\beta$ increases. As $\eta_{SE}$ is a concave function based on **Proposition 1** and $P_T$ is an affine function of $\beta$, so the fractional form $\eta_{EE}$ is quasi-convex in $\beta$ for $\beta \in [0, +\infty)$. Thus, the proof of **Proposition 3** is complete.

**REFERENCES**


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